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# On predicting the interaction of crack-like defects in ductile fracture

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## ABSTRACT

Closely-spaced cracks in structures can interact with each other; the presence of one crack can change the strain energy release rate at another crack nearby. Since this interaction is enhanced by the onset of plasticity, elastic analysis alone should not be used for judging whether interaction between cracks will have a significant effect on the integrity of a structure.

## 1. Article body

A problem encountered frequently in structural integrity assessment is the need to assess structures containing multiple crack-like defects. In pipes and pressure vessels, closely-spaced crack-like defects can occur due to hot cracking of welds or from progressive modes of crack growth such as stress corrosion cracking or fatigue. Integrity assessment procedures including BS7910 [1], R6 [2], ASME B&PVC Section XI [3] and API 579-1 [4] contain criteria for determining whether a set of defects will interact with one another. In an assessment, the ability to show that the initiation of fracture at one defect is not affected by the presence of others can be very beneficial: it removes the need to conservatively re-characterise the crack system as a single enclosing crack or to model the interaction between cracks explicitly.

Criteria for determining whether this type of interaction is significant for fracture initiation are typically established using linear elastic fracture mechanics. Specifically, they may be based on linear-elastic modelling of interacting pairs of cracks and/or experimental observations of fatigue crack growth [5–7]. Normally, it is assumed that the maximum stress intensity factor which occurs anywhere on either crack tip line is the critical parameter for fracture initiation. When the proximity of two cracks increases this maximum stress intensity factor by an ‘unacceptable’ amount, 10% for example, interaction is judged to have a significant effect. The critical stress intensity factor can be affected by the size, shape, relative position and loading of the cracks [8], [9]. However, simple and conservative rules for acceptable crack proximity can be formulated based on results for selected crack pairs.

For a component subjected to a given set of constant surface tractions (i.e. ‘primary’ load [2]), interaction causes a proportionally larger enhancement of the Strain Energy Release Rate (SERR) under elastic-plastic conditions than in material that is purely elastic:

$$\left. \frac{G^{int}}{G} \right|_{el.-pl.} \geq \left. \frac{G^{int}}{G} \right|_{el.} \quad (1)$$

where  $G$  and  $G^{int}$  are SERRs for single and interacting cracks respectively, and the subscripts  $el.$  and  $el.-pl.$  denote elastic and elastic-plastic material respectively. This can alternatively be expressed in terms of crack tip field parameters (pure Mode I loading assumed):

$$\left. \sqrt{\frac{J^{int}}{J}} \right|_{el.-pl.} \geq \left. \frac{K_I^{int}}{K_I} \right|_{el.} \quad (2)$$

where  $J$  and  $J^{int}$  are J-integrals for single and interacting cracks respectively, while  $K_I$  and  $K_I^{int}$  are their stress intensity factors. When interaction criteria are formulated using linear elastic analysis but applied to cracks in elastic-plastic materials, there is a risk that the effect described by Equation (2) could cause unintentional non-conservatism. This is illustrated in the following example, representing a typical situation in which interaction criteria are employed.

Consider the cylindrical pipe with an inner diameter of 400 mm and an outer diameter of 500 mm shown in Fig. 1a. It is assumed to be long, but with closed ends. The pipe contains a pair of identical and co-planar internal semi-elliptical surface cracks in the axial-radial plane. The cracks each have a depth  $a$  of 25 mm and an overall width  $2c$  of 100 mm, giving an aspect ratio  $\frac{a}{c}$  of 0.5. Free from any residual stress or thermal gradient, the pipe is subjected to increasing internal pressurisation which causes both hoop and axial stress in the pipe wall; pressure also acts on the faces of the internal cracks. The pipe material's stress-strain curve follows a Ramberg-Osgood relationship [10] approximating the properties of A533B Class 1 ferritic pressure vessel steel at 20 °C [11] (Young's modulus:  $E = 210$  GPa, Poisson's ratio:  $\nu = 0.3$ , yield stress:  $\sigma_0 = 450$  MPa, hardening exponent:  $n = 7.6$ , yield offset parameter:  $\alpha = 0.9333$ ). It follows incremental plasticity theory with a von Mises yield locus and an isotropic hardening law. Here,

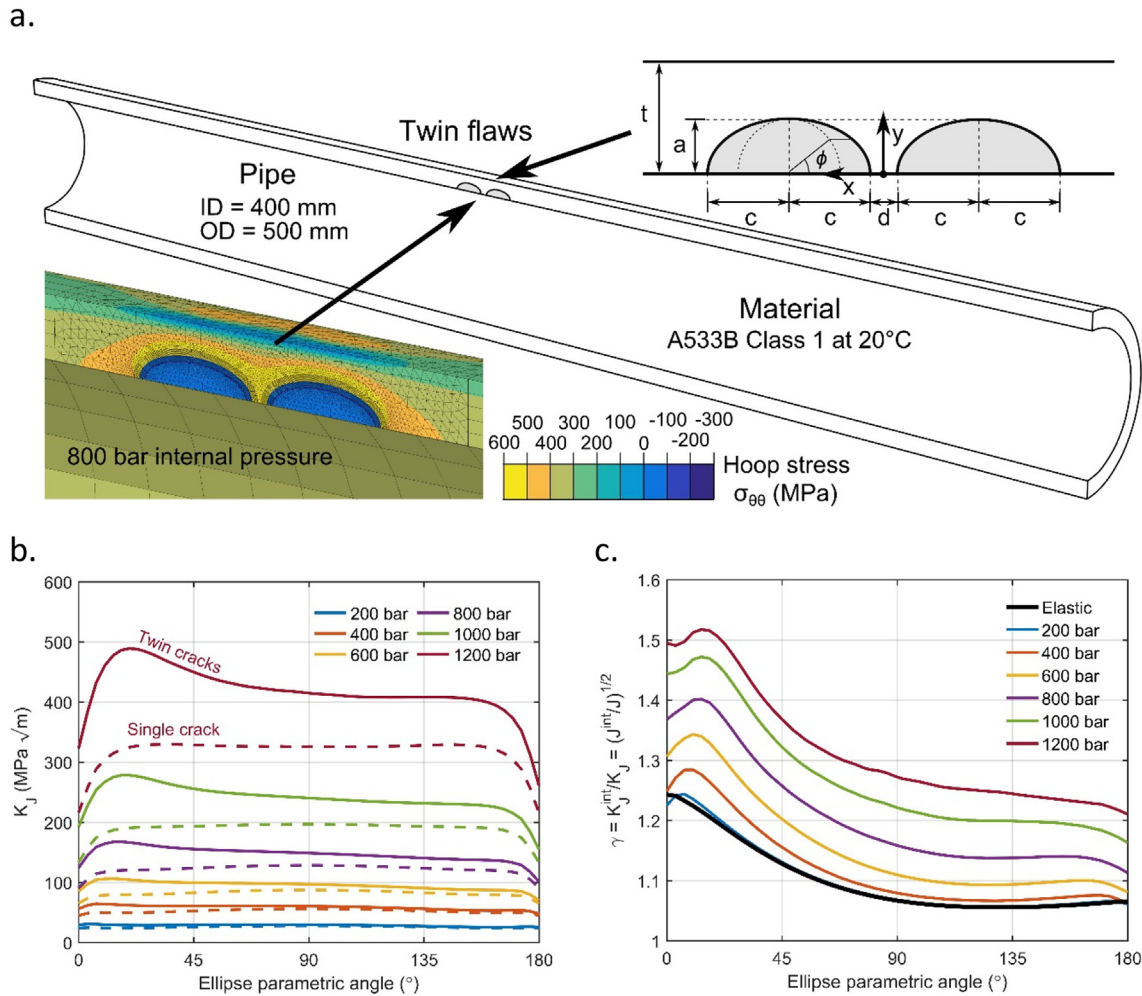
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**Fig. 1.** a.) Analysis of a pressurised pipe containing twin internal surface cracks. b.) Elastic-plastic equivalent stress intensity factor  $K_J = \sqrt{\frac{JE}{1-\nu^2}}$  for a single crack and for twin cracks separated by  $d = 0.25c$  (i.e. 12.5 mm). c.) Interaction factor.

finite element analysis was used to determine the deformation of pressurised pipes containing single cracks and twin cracks and hence to calculate the J-integrals for each case. Limit load analysis was also performed using a rigid-plastic material, also with  $\sigma_0 = 450$  MPa and a von Mises yield locus.

Fig. 1b & c illustrate that two closely-spaced cracks in the pipe wall produce more severe crack tip loading than a single crack. Furthermore, the cracks experience more strongly under increasing levels of applied pressure as the material between them experiences increasing plasticity. This conflicts with earlier suggestions that the interaction effect is either unaffected or weakened by the onset of plasticity [12–14], but agrees with the prediction by Xuan et al. of enhanced interaction under creep conditions [15]. A ‘global’ interaction factor, i.e. the increase in the maximum crack driving force anywhere on the crack tip line, can be defined as [9]:

$$\gamma^G = \frac{\max_{\phi} K_J^{int}(\phi)}{\max_{\phi} K_J(\phi)} = \sqrt{\frac{\max_{\phi} J^{int}(\phi)}{\max_{\phi} J(\phi)}} \quad (3)$$

where  $\phi$  denotes the position on the crack tip line (see Fig. 1a). For the twin-flawed pipe under internal pressure in Fig. 1, the global interaction factor is 1.107 at 200 bar of internal pressure, but increases to 1.484 at 1200 bar. The maximum SERR occurs along the region of the crack tip line closest to the other crack (eg. at  $\phi = 19.7^\circ$  for 1200 bar)

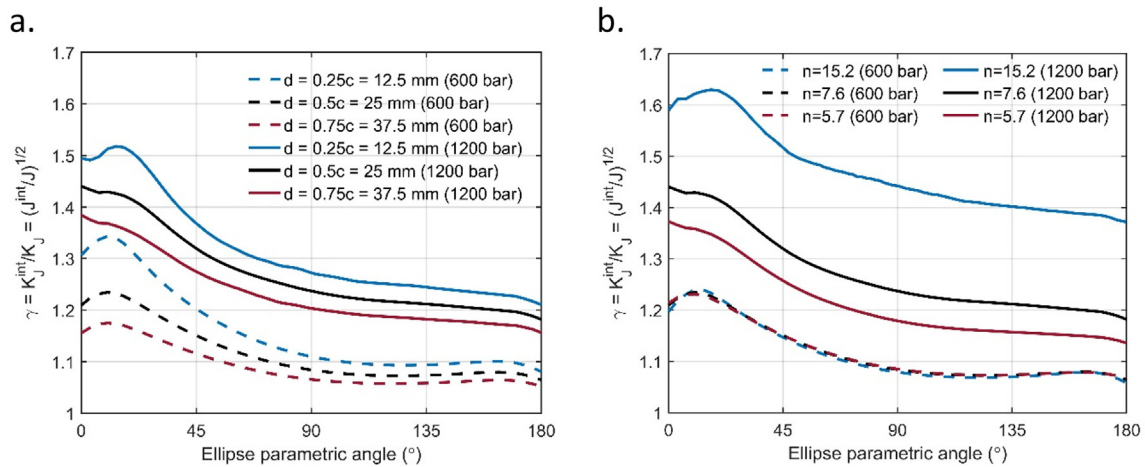
indicating that in a ductile material, tearing would initiate close to the internal surface and cause the cracks to coalesce as the internal pressure is increased. This is consistent with the experimental results of Bezensek and Hancock [16] which showed that in twin co-planar surface cracks under three-point bending, ductile tearing consistently initiated in the ‘re-entrant’ region between them. Closer proximity of the cracks to one another and increasing material hardening increase the interaction under inelastic conditions as shown in Fig. 2.

The observation that stronger crack interaction occurs under elastic-plastic conditions than under linear elasticity is significant for the formulation of interaction criteria. For example, the criterion used to judge significant interaction for this crack geometry in BS7910 and R6 is:

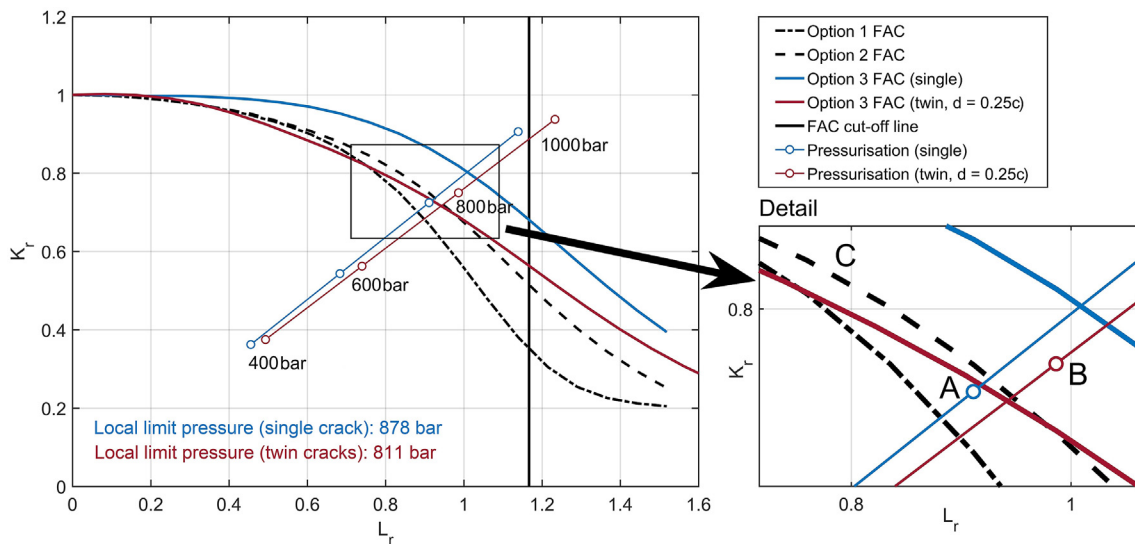
$$d \leq \min(2c_1, 2c_2) \quad \text{for} \quad \frac{a_1}{c_1} > 1 \quad \text{and} \quad \frac{a_2}{c_2} > 1$$

$$d \leq \max\left(\frac{a_1}{2}, \frac{a_2}{2}\right) \quad \text{for} \quad \frac{a_1}{c_1} \leq 1 \quad \text{and} \quad \frac{a_2}{c_2} \leq 1 \quad (4)$$

where  $a_1$ ,  $c_1$  and  $a_2$ ,  $c_2$  are the depth and half-width of the first and second cracks respectively, and  $d$  is the inter-crack spacing. The example shown in Fig. 1 lies at the limit of this criterion. For a wider crack spacing  $d$ , the cracks would not be re-characterised as a single large crack or be subjected to detailed analysis. Subsequent steps in the integrity assessment could be carried out considering the two defects individually. In this situation, unintentional non-conservatism could arise: firstly because the defects’ elastic-plastic interaction generally



**Fig. 2.** Interaction between crack-like defects in inelastic material is affected by the crack spacing and material hardening characteristics. a.) Interaction factor for twin cracks in the pressurised pipe described above, with different spacing of the twin cracks. b.) Interaction factor for twin cracks spaced at  $d = 0.5c = 25$  mm, in materials with different hardening moduli.



**Fig. 3.** Failure assessment diagram for the pressurised tube of Fig. 1. Blue lines are for a pipe containing a single defect with the assessment point at  $\varphi = 90^\circ$ , as would be assumed if the twin defects were judged to be non-interacting. Red lines are for a pipe containing twin defects with the assessment point at  $\varphi = 19.7^\circ$ , i.e. the expected location of fracture initiation. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

raises the SERR, and secondly because the assessment point which would be used in the case of a single crack ( $\varphi = 90^\circ$ ) is not the location of greatest SERR when the cracks are interacting (see Fig. 1b).

Fig. 3 shows a Failure Assessment Diagram (FAD) for the pipe in Fig. 1, following the R6 procedure [2] and assuming  $K_{IC} = 153$  MPa  $\sqrt{m}$ . It compares two cases: the assessment that would be performed if the defects were judged to be ‘non-interacting’ (in blue), and the result from detailed analysis of the crack pair (in red). Comparing points A and B (both for 800 bar pressurisation) illustrates the potential for a non-conservative assessment: they lie on opposite sides of the Option 2 Failure Assessment Curve (FAC) and so a component assessed at Point A assuming non-interaction of defects could in fact be undergoing tearing at Point B. Option 3 FACs are also plotted, using results from J-integral analysis and with plastic collapse parameter  $L_r$  defined using the local limit pressure. The Option 3 FAC for twin cracks lies inside Option 2 across part of its range (see Region C), suggesting that this definition of limit pressure is not conservative for considering fracture initiation in the area in-between the two defects.

It can be argued that potential non-conservatism in Equation (4)

under ductile conditions is offset by conservatism in other aspects of an assessment, specifically:

1. A rising material J-R curve might cause tearing to arrest after a small amount of crack growth.
2. Constraint loss in the region in-between the defects would inhibit fracture here [17].
3. Estimates of the limit load used in a FAD analysis might provide some conservatism (although this was not the case in Fig. 3).

But while it may be acceptable to treat some defect pairs as ‘non-interacting’ under ductile conditions despite the increase in SERR caused by their interaction, this requires careful justification. In summary, elastic analysis alone should not be used to judge whether interaction between adjacent crack-like defects will affect the initiation of ductile fracture. The co-planar surface flaw interaction criteria of BS7910 and R6 are based on elastic analysis and they perform well for predicting elastic interaction [9]. However, these criteria do not conservatively predict the initiation of ductile tearing in all realistic cases.

## Data statement

Data supporting the findings presented in this article may be downloaded from: <https://doi.org/10.5523/bris.1qodyli8i6ajt1yiaycfsoqdbb>.

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